Numerical Analysis of the Power Balance of an Electrical Machine With Rotor Eccentricity

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The power balance in a cage induction machine with eccentric rotor has been studied. The asymmetrical air gap flux-density distribution caused by the non-uniform air gap due to eccentricity produces forces that play an important role in the rotordynamic stability. These forces act both in the radial and the tangential directions. The forces together with the whirling motion produce additional power in the shaft. It is shown that if the power balance of the machine is satisfied, the power due to the whirling can be calculated from the power balance. This also gives a new approach to compute the forces due to eccentricity.

Index Terms— Cage induction machine, eccentricity, electromagnetic torque, finite element method, power balance, rotordynamics

I. INTRODUCTION

The non uniform air gap caused by the eccentricity creates an asymmetrical air gap flux distribution which produces forces, Figure 1. Rigorous studies have been already done in the past to study the eccentricity and the forces [1, 2]. The unbalanced magnetic pull caused by the forces has been a topic of investigation in the past [3, 4]. The analytical models have been validated by the experimental investigations. Conventionally, there are two types of rotor eccentricity. First, the static eccentricity in which the rotor is shifted from the center point but does not make any whirling motion and second, the dynamic eccentricity in which the rotor also makes a whirling motion that is, rotating around the center point at certain frequency and radius called the whirling frequency (ω_w) and the whirling radius respectively. Most of the literature focuses on eccentric cage induction machines and the forces and vibrations related to it. In the case of the dynamic eccentricity, when the rotor is whirling, the velocity vector of the whirling motion combine with the force to produce a power. This power is very small but plays an important role in the rotordynamics and also in the power balance of the machine. In this paper we study the power due to the whirling from the power balance approach. The method based on the power balance has also been used to calculate the torque of an induction machine [5].

The power balance of an eccentric machine should also take into account the power due to the whirling motion. It is shown in the paper that if the power balance of the machine is satisfied at each time step, the power due to the whirling can be calculated from it. This will also give a new approach to verify the force computation methods or calculate the forces due to eccentricity.

II. METHOD OF ANALYSIS

The power balance of a normal healthy machine can be given as

$$P_{\text{in}} = P_{\text{loss}} + \frac{dW_{\text{f}}}{dt} + P_{\text{shaft}} \tag{1}$$
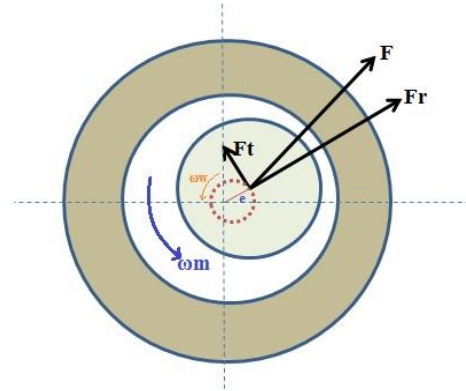

Fig. 1. Forces due to eccentricity. F_r and F_t are the radial and the tangential component of the force respectively

where P_{in} is the input power, P_{loss} is the electromagnetic loss, W_{f} is the energy of the electromagnetic field and P_{shaft} is the shaft power. In case of an eccentric machine we have seen that the P_{shaft} calculated from the power balance increases. So if P_{shaftEcc} is the total power transmitted through the shaft, it should include the power transmitted by the torque (T) plus the power due to the whirling. The power due to the whirling can be calculated by the dot product of the force and the velocity vector such that

$$P_{\text{shaftEcc}} = T\omega_{\text{m}} + P_{\text{whirl}} \tag{2}$$

$$P_{\text{whirl}} = \mathbf{F} \cdot \mathbf{v}. \tag{3}$$

So, the power balance of the eccentric machine can be given as

$$P_{\text{in}} = P_{\text{loss}} + \frac{dW_{\text{f}}}{dt} + T\omega_{\text{m}} + \mathbf{F} \cdot \mathbf{v}. \tag{4}$$

The power balance of (4) should hold at any instant of time. If so, the power balance can be used to calculate the forces due to eccentricity by calculating the power due to whirling. For this purpose, we check the validity of the power balance by studying a case when the rotor is eccentric but does not make any whirling motion.

A time stepping finite element analysis is performed. One

period of the fundamental frequency is divided into 800 time steps and second order triangular elements are used for the study. The whole mesh has 4290 elements and 8677 nodes. The magnetic field in the core region of the machine is assumed to be two-dimensional. The three dimensional end winding fields are modeled approximately by adding the end-winding impedances to the circuit equations of the windings. The nonlinearity of the materials is taken into account by using the single valued magnetization curve.

The center position of the rotor is forced to move along a circular path at a constant speed to model the dynamic eccentricity. In addition, the rotor was rotated in its mechanical angular speed. The movement of the rotor is taken into consideration by the moving-band technique.

The forces related to the eccentricity can be calculated by using the Coulomb's method [6] that is based on the principle of virtual work and the same method is used to calculate the torque. The expressions for the force and torque are

$$ F_{x,y} = \frac{l}{\mu_0} \sum_{e=1}^{N_{\text{cag}}} \int_{\Omega_e} \left[-\mathbf{B}^{\mathrm{T}} \mathbf{G}^{-1} \frac{\partial \mathbf{G}}{\partial x, y} \mathbf{B} + \frac{1}{2} B^2 |\mathbf{G}|^{-1} \frac{\partial |\mathbf{G}|}{\partial x, y} \right] d\Omega \quad (5) $$

$$ T = \frac{l}{\mu_0} \sum_{e=1}^{N_{\text{cag}}} \int_{\Omega_e} \left[-\mathbf{B}^{\mathrm{T}} \mathbf{G}^{-1} \frac{\partial \mathbf{G}}{\partial \varphi} \mathbf{B} + \frac{1}{2} B^2 |\mathbf{G}|^{-1} \frac{\partial |\mathbf{G}|}{\partial \varphi} \right] d\Omega . \quad (6) $$

III. RESULTS AND DISCUSSIONS

A 4-pole 15-kW cage induction machine is used as a test machine in this study. The machine was simulated for different values of eccentricity. First, the dynamic eccentricity was studied. The forces were calculated and thus the power due to the whirling P_{whirl} was obtained. The whirling power is shown in Figure 2.
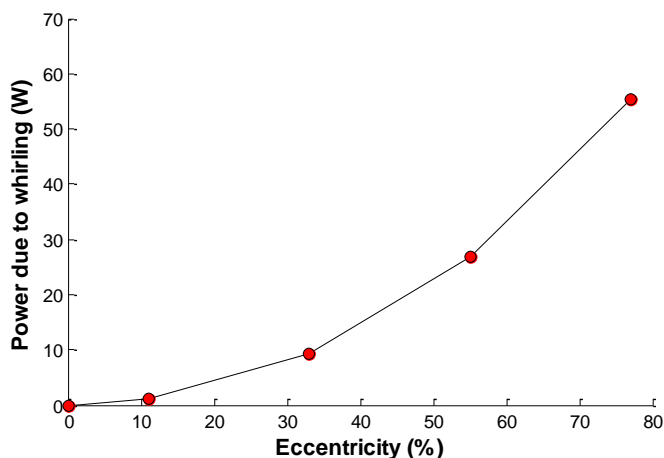
Fig. 2. Power due to whirling with respect to the eccentricity

Now, if the machine is simulated with static eccentricity, which means that there is no whirling motion and the power related to it, the error in the power balance can be calculated. The power balance error strongly depends on the time integration method used in the simulation. In this paper

Implicit Euler and Trapezoidal rule are tested for the power balance. The errors in the power balance of the machine are shown in Figure 3. The Implicit Euler shows large error. When Figures 2 and 3 are compared, it can be clearly seen that the power due to whirling is less than the error (except for ε = 77%). This suggests that the whirling power cannot be calculated from the power balance when the Implicit Euler is used. For the Trapezoidal rule, the errors are much less than that from the Implicit Euler. Again comparing Figures 2 and 3, we see that the error is less than the power due to the whirling. So, we can say that the power due to the whirling motion can be calculated from the power balance if the Trapezoidal rule is used in the simulations for the time integration. Similarly, from the whirling power the forces due to the eccentricity can also be computed.

In the full paper, the numerical power balance will be discussed in detail and the calculation of the whirling power and thus the forces due to eccentricity from the power balance of the machine will be shown and validated by the measurements.
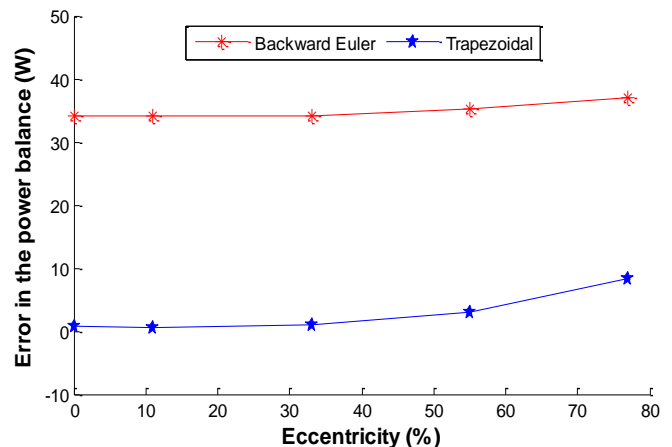
Fig. 3. Error in the power balance when rotor is eccentric but no whirling motion exists

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